THE DYNAMIC COMPUTER-SUPPORTED VISUALIZATION AMPLIFYING STUDENTS' GREATER AWARENESS OF THE CONCEPT OF LIMITS IN THE COURSE OF FURTHER MATHEMATICS

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Abstract

Objectives: The aim of the research is to make students' perception of the concept of limits better in the course of Further Mathematics based on a set of teaching and learning aids consisting of a workbook and computer animation.

Methods / Analysis: A mathematics course that has been taught at university in the technical major is intended to provide the necessary basis for studying other vocational subjects. However, many students find the course of Further Mathematics one of the most difficult to master. The reasons for this belief are a lower number of class hours and a much-decreased level of secondary school mathematical proficiency of first-year students. The lecturers had to develop a module of teaching methods that contained a workbook and dynamic computer presentations (in the form of a slide show) aimed at mastering students' maths skills. The authors chose the topic devoted to "The concept of limits" for the experiment. Computer-supported visualizations were offered in Maple and PowerPoint. The lecturers gave classes using a teaching-aid set for students majored in Electrical Power Engineering and Electrical Engineering, Construction, Information Systems and Technologies at Sholom-Aleichem Priamursky State University during the 2016-17 and 2017-18 academic year. The survey involved 62 first-year students (there were 34 students in a test group and 28 students in an experimental one). The students were to take a final questionnaire after having finished studying the topic.

Findings: The results of the survey indicate that the classes with specially designed teaching aids (workbooks and computer visualizations) proved to be more effective and dynamic. 91% of questioned students held a positive view towards the workbooks. The students answered the question "How much animation video clips have helped in theoretical studying the concept of limits according to a 0 to 5 grading scale (where 5 is excellent, 0 is no)" in the following way: "5" - 68.2%; "4" - 13.6%; "3" - 9.1%, "2" - 9.1%. As a result of the educational experiment, the average rating of students' academic progress in the experimental group is 37% higher than that in the test group. Using the given teaching aids in the educational process made it possible to have the following advances: (a) to free up some of the lecture time; (b) to add value of students' cognitive activity via information technology training; (c) to work out basic skills and abilities; (d) to differentiate the educational process, providing individual opportunities for students who are interested in mathematics to do more complex assignments; (e) to drill and practise the English mathematical vocabulary, using definitions in the presentation not only in Russian, but also in English; (d) to create positive learning experience in the subject, involving entertaining or little-known historical facts. The use of abstract mathematical notions in the presentation of dynamic computer visualization allows introducing them to students in a flexible way for perception, thus avoiding any formalism of knowledge.

Applications / Improvements: These days, the scholars have been developing similar teaching and learning aids organized thematically on sections Derivation and Integration.

Keywords: computer animation, workbook, concept of limits, teaching and learning aids, dynamic computer-supported visual representation (visualization).

1 INTRODUCTION

A lot of researchers' surveys focus on challenges students face in mastering the basic concepts that constitute calculus. And the most difficult is the concept of a limit [1, 2, 3]. It is caused by a semantic gap between a definition expressed in formal terms of a limit and intuitive knowledge of the meaning [4, 5, 6]. The scientists identify three forms of barriers to students' understanding a limit. They are epistemological barriers related to mathematical roots of a limit; cognitive barriers connected to the abstract nature of a limit; didactic barriers referred to mathematical methods [7].

Several authors' studies submit that students usually have a number of common misconceptions when studying the theory of limits [8, 9]. M. Przenioslo' research is devoted to the definition of students' images of the concept of a limit, clarifying their associations related to the limits, and determining the efficiency level. The study started because the scholar was dissatisfied with the students' understanding the concept of a limit, on the one hand and on the other hand, his own interest to improve teaching it both at secondary school and university [10].

J.Tertia' thesis research explores revealing the wrong ideas of engineering students about a limit. The scientist suggests "changing the out-of-date method of spoken language to a more modern approach that allows students making conclusions by themselves and developing their own comprehension of limits; solving problems and doing team work". In his point of view, students should "study the effect of using technology, for example, software applications and graphing calculators" [9] to understand the theory of limits better.

I. Kidron and N. Zehavi's study observes their survey of using animation as a means of improving learners' understanding the concept of a limit in the *Approximation and Interpolation* course. Students developed dynamic graphics via Mathematica software that both visualized the process of the approximation of functions by Taylor polynomials and assigned a specific meaning to the definitions [11].

The aim of the research is to make students' perception of the concept of limits better in the course of Further Mathematics based on a set of teaching and learning aids consisting of a workbook and computer animation

2 METHODOLOGY

The authors produced learning and methodological package. It consisted of workbooks and slide shows on two topics 'The Limit of a Sequence' and 'A Limit and Continuity of function'. Computer animation was developed by means of Maple (a computer algebra system) and MS PowerPoint.

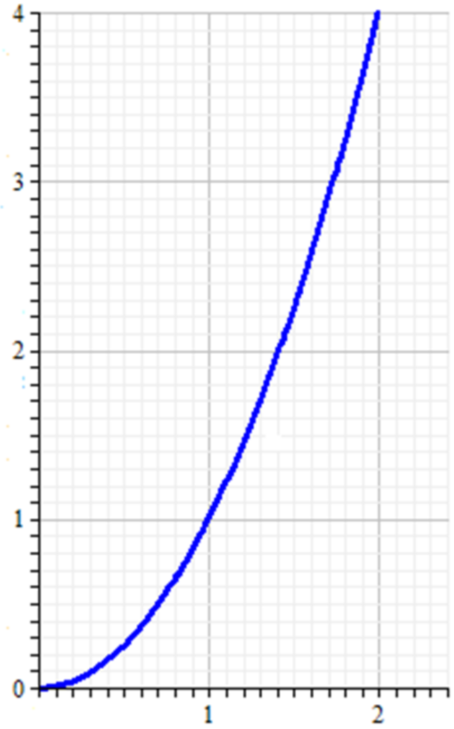
Here is an example of the topic 'A Limit and Continuity of function' from the learning and methodological package. The concept of a limit touches upon one of the most important concepts of calculus. The definitions of a continuous function at a point, the derivative of a function and a definite integral are made up through the limits of originally generalized functions. But most first-year students cannot master the concept of a limit properly because this notion is too abstractive (conceptional). Therefore, the introduction of an exact definition of the limit of a function is preceded by the following practical task (Fig. 1) in the original workbook that the authors had developed.

At the same time, formulae are displayed in PowerPoint slides that specify three sequences of points:

$$1)\ x_n = 1 + \frac{1}{2^{n-1}};\ 2)\ x_n = 1 - \frac{1}{2^{n-1}};\ 3)\ x_n = 1 + \frac{(-1)^n}{2^{n-1}}.$$

Each student is supposed to select one of the formulae and give the limit of the selected sequence. After that, students have to fill in a list of values x_n and $f(x_n)$ for $n = 1, 2, ..., 6$.

Example 1. Consider the function $f(x)=x^2$.



We choose a sequence of points

$x_n =$

We choose a sequence of points

$\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} \qquad =$

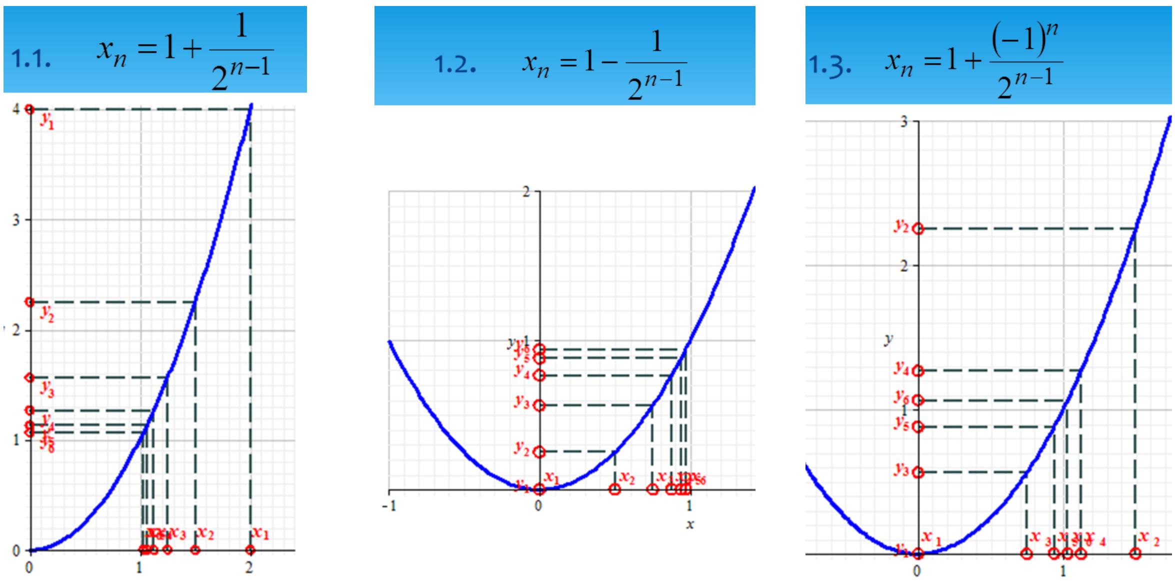
n	x_n	$y_n = f(x_n)$
1		
2		
3		
4		
5		
6		
…	…	…

We calculate the limit of a sequence of values of a function at points

$\lim\limits_{n\to\infty} f(x_n) = \lim\limits_{n\to\infty} \qquad =$

Figure 1. The workbook Task, preceding the introduction of the definition of the limit of a function by Heine.

Then, they are offered to view three animations, where a graph of the function $f(x) = x^2$ is displayed on the coordinate plane, and points that fade into view that define the calculated first elements of sequences x_n on the abscissas and $y_n = f(x_n)$ on the ordinate (Fig. 2). The dependence of the variable y on the argument x is emphasized by dotted graph – the verticals, directed from the point of the function graph to the corresponding points on the coordinate axes.

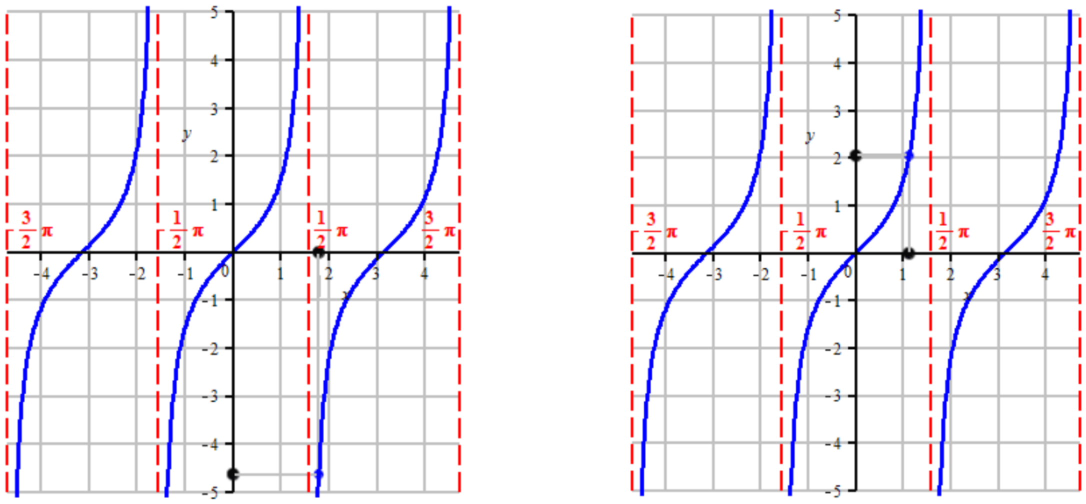


a) $x_n \to 1$ on the right; b) $x_n \to 1$ on the left; c) $x_n \to 1$ from both sides;

Figure 2. Animation frames showing unbounded point approximation x_n and points $y_n = (x_n)^2$ at their limit points.

Such a computer-generated visual system helps students to make a conclusion: it follows $\lim_{n\to\infty} f(x_n)=1$ because $\lim_{n\to\infty} x_n=1$ (although, all three sequences proposed tend to limit in a different way!!!). And in mathematical notation, this is written as: $\lim_{x\to 1} x^2=1$. After completing this assignment, students are prepared to be aware of the exact definition of the limit of a function by Heine.

The authors used Maple computer algebra system to create animated clips that visualized the definition of the limit of a function. Besides the limit of a function that had already outlined above, the animations of the limits $\lim_{x\to\frac{\pi}{2}+0} \operatorname{tg} x=-\infty$, $\lim_{x\to\frac{\pi}{2}-0} \operatorname{tg} x=+\infty$, $\lim_{x\to+\infty} \operatorname{arctg} x=\frac{\pi}{2}$, $\lim_{x\to-\infty} \operatorname{arctg} x=-\frac{\pi}{2}$ and many others have been produced (Fig. 3,4).



a) in $x\to\pi/2+0$ b) with $x\to\pi/2-0$

Figure 3. Animation one-sided (single-ended) limits of a function $f(x)=\operatorname{tg} x$.

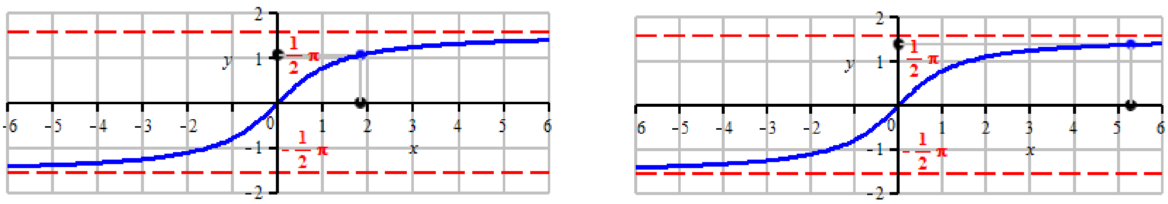
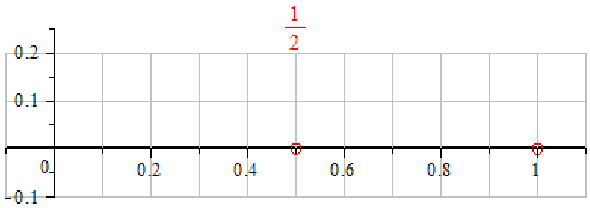


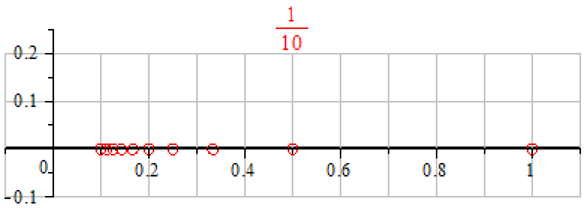
Figure 4. Limit animation frames $\lim_{x\to+\infty} \operatorname{arctg} x=\frac{\pi}{2}$

The presentation of the fact that the limit of a function $f(x)=\sin x$ when $x\to+\infty$ does not exist was also given through that kind of animation clip.

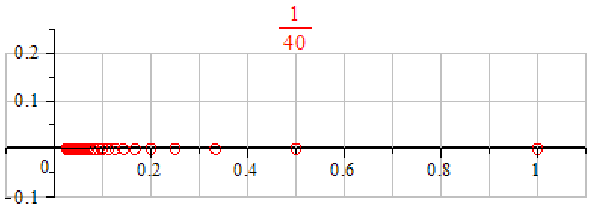
The authors also visualized the concept of the limit of a numerical sequence using Maple. Students could observe the successive approximation of the points $\{x_n\}$ to the limit (Fig. 5) while looking through the animation clips.



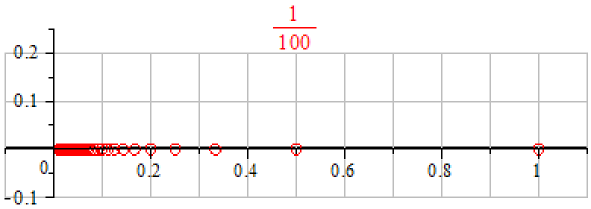
a) $n = 2$



b) $n = 10$



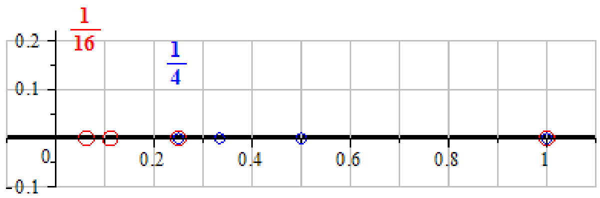
c) $n = 40$



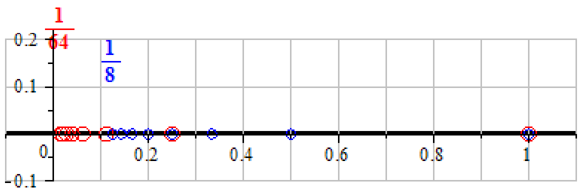
d) $n = 100$

Figure 5. The animation clip frames showing the elements of the sequence that eventually get arbitrarily close to zero. $\{1/n\}$.

After watching the animation clips showing the sequence that eventually get arbitrarily close to zero $\{1/n\}$, $\{-1/n\}$, $\{(-1)^n/n\}$, it is getting possible to introduce the notion of inequilateral limits. And a similar animation for sequences $\{1/n\}$ and $\{1/n^2\}$ visualizes a different degree of convergence (Fig. 6).



a) $n = 4$



b) $n = 8$

Figure 6. The animation clip frames showing the rate of sequence getting close to zero. $\{1/n\}$ *and* $\{1/n^2\}$.

3 RESULTS

The experiment was carried out at Sholom-Aleichem Priamursky State University in the academic year 2016-2017 and 2017-2018. Mathematical Analysis classes were taught to the first-year students majoring in Construction Engineering and Electric Power Engineering and Electrical Engineering (test group involved 34 people) by a traditional approach. The first-year students majoring in Information Systems and Technologies (experimental group numbered 28 people) had classes with original workbooks and dynamic computer presentations applied.

After completing the Mathematical Analysis course in fall semester, the students' learning points (calculated in rating scores) in each group were compared by the nonparametric Rosenbaum Q-test for unconnected samples. The null hypothesis H_0 was tested. The students' scores studying Mathematical Analysis via the dynamic computer visual tools are turned out to be equal to the students’ scores studying Mathematical Analysis by a traditional approach. Supplied by the competing H_1 hypothesis, the amount of scores of students studying Mathematical Analysis with dynamic computer imaging is more than those ones’ who studies Mathematical Analysis by means of traditional methods.

Drawing a parallel between the determined criterion empiric value Q_{emp}=11 and the critical value Q_{cr}=10 ($Q_{emp} > Q_{cr}$) with the sample n_1 =28, n_2=34, the authors conclude that an alternative hypothesis is accepted at the significance level of 0,01. Hence, the scores of students who study Mathematical Analysis by dynamic computer visualization are superior to the scores of students studying Mathematical Analysis in a more common way.

The comparison of the points scored by the students in the test and experimental groups for the current semester activity makes it possible to sum up that the application of dynamic computer presentations is productive for studying Mathematical Analysis.

The students in the experimental group were offered a questionnaire. The outcomes proved the classes where the lecturers applied original workbooks and presentations were more effective and dynamic.

The findings of the questionnaire survey testify that classroom activities by the aid of workbooks and presentations turned out to be more active and effective. 91% of the respondents answered the question “Do you find a workbook necessary?” affirmatively. In answer to the question “What is a better way to be given a lecture to you either with a PowerPoint presentation applied or just verbally?” 87% of the participants voted for the presentation. 91% of interviewees replied to the question “Do the classes motivate you to be interested in Math’s?” positively. 4,5% of students said, “Probably” and 4,5% of students said, “Not always”. In addition, students were asked to rate the quality of animation clips from 0 to 5 points: where 5 is excellent, but 0 is unsatisfactory. The scores were classified in the range as follows: 5 – 86,4%; 4 – 9,1%; 3 – 4,5%. Evaluating how much valuable the animated movies were when realizing the definition of a limit according to the grading scale from 0 to 5 (5 – they were very helpful, 0 – had no use at all), whether the clips made the definition clearer after watching the video, the students gave the following answers: 5 – 68,2%; 4 – 13,6%; 3 – 9,1%, 2 – 9,1%.

4 CONCLUSIONS

The use of such workbooks and dynamic presentations in training made it possible to achieve the following aims:

- to free up some time dedicated to a lecture;

- to promote students’ cognitive activity through information technology training.

The lecturers were able to cover a number of issues from the time saved:

- to master basic skills;

- to differentiate the academic activity, supplying students interested in mathematics with advanced level assignments;

- to drill and master the English mathematical terms, using definitions in the presentation not only in Russian, but also in English;

- to heighten students’ interest in the subject, including entertaining or out-of-the-way historical items of information.

The use of abstract mathematical concepts in dynamic computer video presentations gives an opportunity to picture them in a more suitable form to students’ perception, so that providing real knowledge.

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