International Journal of Modeling, Simulation,
and Scientific Computing
Vol. 6, No. 2 (2015) 1550012 (9 pages)

DOI: 10.1142/S1793962315500129

The magnetic states and Labyrinth ordering in 2D lattices of ising spins with RKKY Interaction

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Received 13 November 2013
Accepted 8 November 2014
Published 4 February 2015

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The numerical simulation of 2D simple square lattice of Ising superspins, that interact via the Ruderman–Kittel–Kasuya–Yosida (RKKY) exchange interaction is performed. It is found, that at low temperatures in the researched systems predominantly realized labyrinth domain structure. The modeled image of magnetization distribution does not coincide with the image of energy distribution. Hysteresis loops of the samples and labyrinth domain structures similarly obtained by Monte Carlo method, are also observed in physical experiments.

Keywords: Ising model; long-range interaction; magnetic states; Monte Carlo simulation.

AMS Subject Classification: 82B20, 03C13, 82B26, 82B27, 82B80

1. Introduction

The spin glass state is realized typically in alloys with magnetic impurities in a nonmagnetic host metal (e.g., Mn in Cu, Fe in Au) interact via the long-range oscillatory Ruderman–Kittel–Kasuya–Yosida (RKKY) magnetic interaction.[1–3] The RKKY coupling — i.e., the exchange interaction between localized core spins mediated by metallic electron gas — has been known for 50 years as the basic interaction in metallic ferromagnets. The oscillatory character of the RKKY coupling causes a spin glass behavior in diluted magnetic metal lattices. It rules the interlayer coupling in magnetic layered structures. As it has been shown for the last few years, the RKKY interaction is also the dominant spin interaction in diluted ferromagnetic semiconductors.[4]

At zero temperature, each spin is locked in a particular direction, but directions of spins are distributed randomly so that conventional ferro- or antiferromagnetic order is absent. Theoretical models suppose, that spins arranged on a regular lattice with interactions are randomly ferro-[5,6] or antiferromagnetic.[7,8]

The modern level of theoretical physics and, in particular, numerical simulation allows us to study the properties of planar nanostructures, such as 2D arrays of spins (superspin). However, the possibility of creating real 2D metals is questionable. References 9–11 describe the facilities of the existence in nature and the creation of real 2D metals as well as the potential to create experimental samples of 2D metals.

Discussed in this paper the results were obtained by the authors by means of Monte Carlo (MC) simulations and analytical calculations.[12–15] In Ref. 13, we have described conditions for the phase transition for a random distribution in a volume of spherical nanoparticles with RKKY interaction. Authors showed in Ref. 15 that the MC simulation systems with RKKY interaction leads to the formation of a complex magnetic structure.

In this paper will be shown, that the obtained by means of a numerical simulation, a solution of the problem of a magnetic state of 2D lattice of Ising superspins with RKKY exchange interaction, suggests that in this system the most probable magnetic states are labyrinth magnetic structures. The processes of a reorientation of magnetization under influence of external magnetic field, and simulation of magnetic hysteresis properties was performed.

1.1. *RKKY interaction in superspin system*

The energy of interaction for effective magnetic moments of spherical particles according to Ref. 16 is

$$E_{3D}(R) = -\frac{J_0}{2}\sum_{i=0}^{N-1}\sum_{j=i+1}^{N} F(x) m^i_{\text{eff}} m^j_{\text{eff}}, \tag{1}$$

$$F(x) = \frac{2x\cos x - \sin x}{x^4}, \tag{2}$$

$$m^i_{\text{eff}} = \frac{\pi M}{2k_f^3}(\sin y - 2x\cos(y)), \tag{3}$$

where S_i, S_j are superspins, M is magnetization, $x = 2k_f R, y = 2k_f R_c, R$ is the distance between particles, R_c is the radius of the spherical particle, k_f is the Fermi wave vector. Under the superspin we mean area joined by RKKY interaction spins, in which the magnetic spins of the atoms rotate coherently. For granules and nanoparticles, the size of superspin is limited by their size.

The energy of the spin system, located in the plane and interacting via RKKY exchange[13,17]:

$$E_{2D}(R) = -\sum_{i=0}^{N-1}\sum_{j=i+1}^{N} A\frac{\sin x}{x^2} S_i S_j, \tag{4}$$

in this case $k_f^2 = 2\pi n_s, n_s \sim \frac{1}{a^2}, a$ is the lattice constant.

1.2. *The MC simulation*

In order to study frustrated spin systems, people use different computational techniques. One of these techniques is the MC method, which is a powerful class of algorithms that is used not only in physics, but also in other fields like engineering, chemistry, biology, material science, etc.[18–20] Although, the obtained dynamics in the MC simulations is intrinsic and the time evolution of the system does not come from any deterministic equation for the magnetization, the results of the MC simulations reproduce qualitatively the trend of the experimental data. Actually, this good qualitative agreement between the simulation results and the experimental data enable us to have a better insight into the nanoscale phenomena, though some of them stem from nonequilibrium processes.[21] For the simulation, we used the Metropolis algorithm.[22]

The Metropolis algorithm steps are as follows

(1) We calculate the interaction energy of the spin with its neighbors in the original position E_1 and in the new one E_2. The energy of the new position was compared with the energy of the old one.
(2) The new position is accepted and becomes the initial for the next step, if $E_2 < E_1$. Otherwise it calculates the probability of reversal p and generate

a random number from the interval $(0, 1)$ (5):

$$p = \begin{cases} 1, & \text{if } E_2 < E_1, \\ \mathrm{e}^{-\frac{\Delta E}{T}}, & \text{if } E_2 > E_1. \end{cases} \tag{5}$$

(3) If p is greater than this random number, then the new position is accepted, otherwise it is rejected, and the old position remains the initial for a new attempt.

The MC simulation technique, with the implementation of the Metropolis algorithm, has been proved a very powerful tool for the systematic study of the magnetic behavior of single nanoparticles and nanoparticle assemblies.[23–27]

2. Comparison of the Results of Numerical and Physical Experiments

The numerical simulation of the system of Ising superspins with RKKY interaction by MC method allows us to obtain the magnetic configuration of the labyrinth domain structure similar to those observed in iron-garnet films.[28,29]

Figure 1(a) shows the magnetization distribution in 2D simple square lattice superspin system with the RKKY interaction. In the system of spins is observed the finite size effect and dependence of the magnetic order of the geometric characteristics of the sample.[25,26] The magnetic landscape of this sample shown in Fig. 1(b). Dark pixels correspond to particles located at the minimum of energy and bright pixels correspond to an excited state of superspins. The interaction energy of each spin with all its neighbors are calculated according to the formula (4).

It was shown that a labyrinth structure (Figs. 1(a) and 2(a)) is the realization of one of the four equiprobable states associated with a fourfold degeneracy of the energy. This is confirmed by fourfold overlay images of the labyrinth structure with each turn of the next image on 90° relative to the underlying one, Fig. 2(b).

Visualization of the magnetization reversal of the sample 50×100 in an external magnetic field is shown in Fig. 3. The magnetization reversal process of the superspins with RKKY interaction begins with the edges of a studied rectangular sample. Thus, the long-range RKKY exchange for selected distances between superspins leads to the appearance of the maximum value of the field is at the edges of the simulated sample. The compensation of the magnetization is carried out by occurrence of the labyrinth state.

The calculated hysteresis loop is shown in Fig. 4. This loop has a qualitative agreement with the experimental data given in the paper.[29,30] It should be noted that in Ref. 30 was performed imposition of hysteresis loops obtained experimentally, and the results of simulation of the Ising spin systems with dipole–dipole interaction, by MC methods, Fig. 4(b). The difference between these results can be explained by, that in Ref. 30 to calculate the total interaction energy, authors

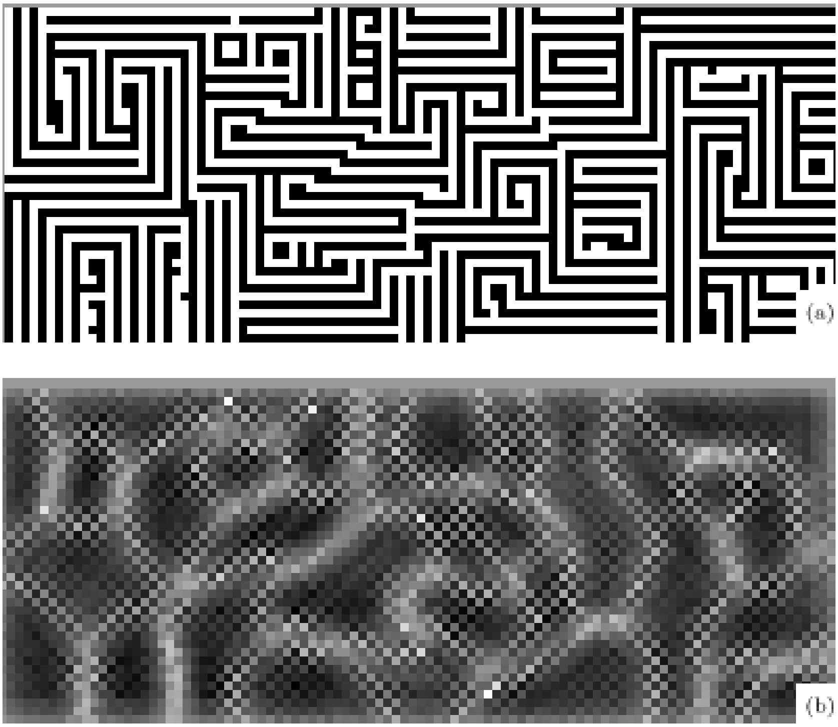
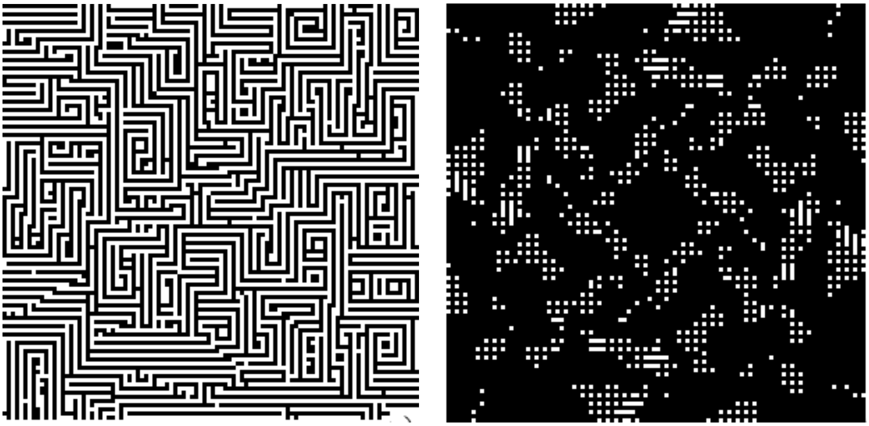


Fig. 1. (a) The magnetic state of system of 40 × 100 superspin. (b) The magnetic landscape for system of 40 × 100 superspin.



(a) (b)

Fig. 2. (a) The system of 100 × 100 superspin. (b) Fourfold overlay images of the labyrinth structure.

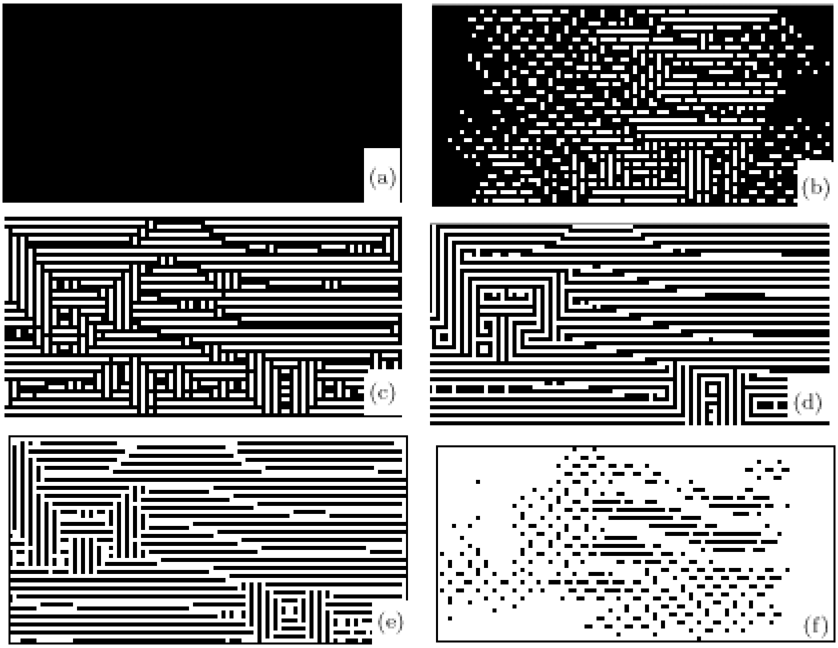
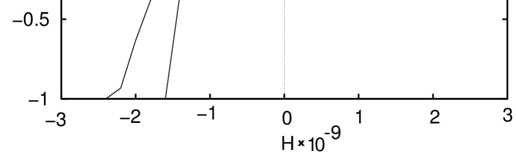


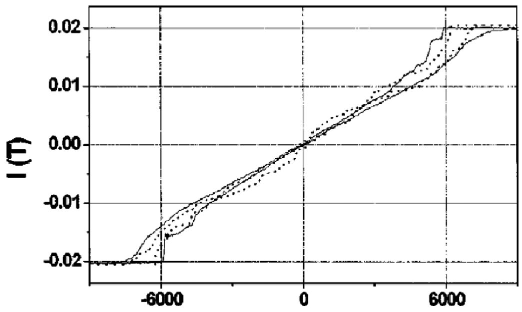
Fig. 3. Visualization of the magnetization reversal of the sample 50×100 in an external magnetic field.

introduced the term, which was responsible for anisotropy energy. A broader hysteresis loop observed by the authors, can be explained by a greater nonequilibrium in the system, i.e., by reducing the number of MC steps in numerical experiments.



(a)

Fig. 4. (a) The hysteresis loop for the sample 50×50 superspins with RKKY interaction. (b) The experimental hysteresis loop.



(b)

Fig. 4. (*Continued*)

At zero field is observed compensation magnetization $M = 0$. In small fields, the susceptibility increases while maintaining the paramagnetic component. When approaching the saturation, the part of macrospins oriented along the field. For small fields, they compensated internal fields on most of the spins, i.e., contributed to frustrations. The internal fields are changed in such a way that their opposition to the external field increases with the external field. Temperature and time are important factors influencing on the hysteresis properties.

3. Conclusions

The observed magnetic state as a labyrinth domain structures in 2D systems of Ising superspins with RKKY interaction located on a simple square lattice, due to the alternating nature of the long-range oscillatory exchange. A key role in the observed in numerical simulations of the magnetic properties of the samples plays a simple square lattice and a set distance on this lattice between the superspins.

The complex magnetic states in the form of labyrinth domain structures are also observed in physical experiments in iron-garnet films.[28–30] It is clear that the observed in numerical experiments the magnetic state is not necessarily a ground state of the system. And of course, that a fourfold degeneracy of the observed state, may have an even higher degree of degeneracy. However, the question of finding the ground state and the multiplicity of its degeneracy for systems with a relatively large number of particles remains open due to the lack of efficient algorithms for solving this problem.

Another open question is the question of the magnetic properties of the studied magnetic systems, which the degeneracy may allow of the existence of states with the maximum value of the spin excess and states with compensated magnetization. The equality of the internal energy of these states determines the equality of the values of the probability of their realization. At the same time the definition of ferromagnetism presupposes the existence only twofold degeneracy of the ground state with the maximum spin excess (in Ising model "all Up" or "all Down"). How the systems of superspins will behave at a high degree of degeneracy, i.e., with the presence of equiprobable states with maximum spin excess and lack of it, may be the subject of further research of the authors.

Acknowledgments

Scientific Fund of Far Eastern Federal University ($\#12-07-13000-18/13$) and the state task of the Ministry of Education and Science of Russia #559, supported this work.

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